Packed Distillation Columns and Absorbers at Steady State Operation

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In this paper the concept of a mass transfer section is applied to problems involving packed distillation columns and packed absorbers. When this concept is employed, the resulting equations required to describe packed distillation columns and packed absorbers are identical in form to those required to describe distillation columns and absorbers with plates. Data from the results of two field tests are presented and analyzed.

Presented for the first time is an exact functional relationship between columns with plates (a stage process) and columns with packing (a continuous process). By use of the concept of a mass transfer section, it is possible to represent a continuous separation process by a stage process that is exactly equivalent.

Although considerable effort has been directed toward the solution of problems involving distillation columns and absorbers with plates (*1*, *5* to *8*, *12*, *14*, *15*), little attention has been given to the improvement of calculational procedures for packed distillation columns and packed absorbers. Beginning in 1923, Whitman, Lewis, Walker, and McAdams (*9*, *10*, *18*, *19*) proposed, developed, and applied the two-film concept to mass transfer operations. Peters (*13*) proposed the use of the concept of the Height Equivalent to a Theoretical Plate (HETP) and Chilton and Colburn (*2*) proposed the use of the concept of the Height of a Transfer Unit (HTU) in the analysis of mass transfer operations.

THE MASS TRANSFER SECTION

The concept of a mass transfer section described by the mass transfer relationship has been used (*7*, *8*) to account for the deviation of an actual plate of a distillation column from a perfect plate. For any plate j, the mass transfer relationship is given by

$$y_{ji} = E_{ji} \gamma_{ji}^L K_{ji}^L x_{ji} \tag{1}$$

where E_{ji} is the vaporization efficiency for component i and plate j, K_{ji}^L is the ideal solution K-value (evaluated at the temperature and pressure of the liquid leaving plate j), x_{ji} and y_{ji} are the mole fractions of component i in the liquid and vapor streams, respectively, leaving plate j, and γ_{ji}^L is the activity of component i in the liquid leaving plate j. The vaporization efficiency E_{ji} is seen to account for the deviation of an actual plate from an equilibrium stage (or a perfect plate for which $E_{ji} = 1$ for all i and a given j) in the same manner that γ_{ji}^L accounts for the deviation of the liquid phase from an ideal solution ($\gamma_{ji}^L = 1$ for all i and a given j).

The concept of a mass transfer section may be applied to packed columns in the following manner. Let the packed portions of the column be divided into sections or elements $\Delta z_1, \Delta z_2, \ldots, \Delta z_N$, where the elements are numbered down from the top of the column as shown in Figures 1 and 2. Let y_{ji} in Equation (1) be the mole fraction of component i in the vapor leaving element Δz_j and x_{ji} the mole fraction of component i in the liquid leaving element Δz_j. Similarly, γ_{ji}^L and K_{ji}^L are evaluated at the conditions of the liquid leaving the element Δz_j. Then E_{ji} becomes that function for each component i and each element Δz_j that is required to relate the mole fraction of component i in the vapor and liquid leaving the element as indicated by Equation (1). It should be noted that if E_{ji} is equal to unity for any one component for a given element of packing, then that particular Δz_j corresponds to the HETP for that component at that particular location in the column. The E_{ji}'s are also related to the overall mass transfer coefficients and HTU's. As discussed in a subsequent section, it can be shown that there exists a unique set of E_{ji}'s which satisfy all of the equations required to describe N increments of a packed column. Corresponding to this set of E_{ji}'s, a set of overall mass transfer coefficients and a set of HTU's may be computed by formulas presented in a subsequent section.

PACKED DISTILLATION COLUMNS

The mass transfer relationship makes it possible to relate the flow rate v_{ji} of each component in the vapor leaving element Δz_j to its flow rate l_{ji} in the liquid leaving Δz_j by use of modified absorption and stripping factors as follows:

$$v_{ji} = S_{ji}\, l_{ji}, \text{ and } l_{ji} = A_{jj}\, v_{ji} \tag{2}$$

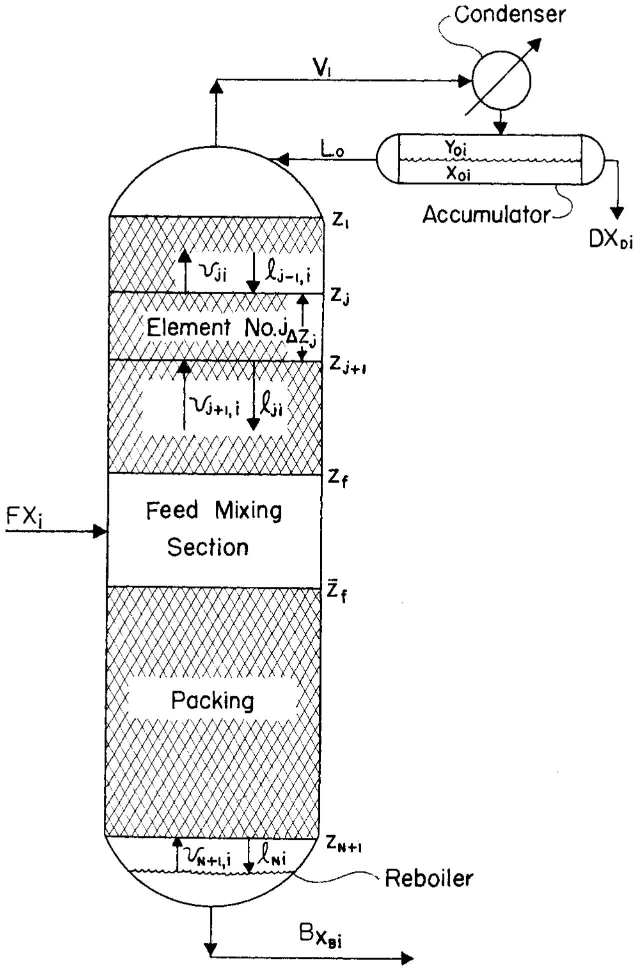

Fig. 1. Sketch of a packed distillation column.

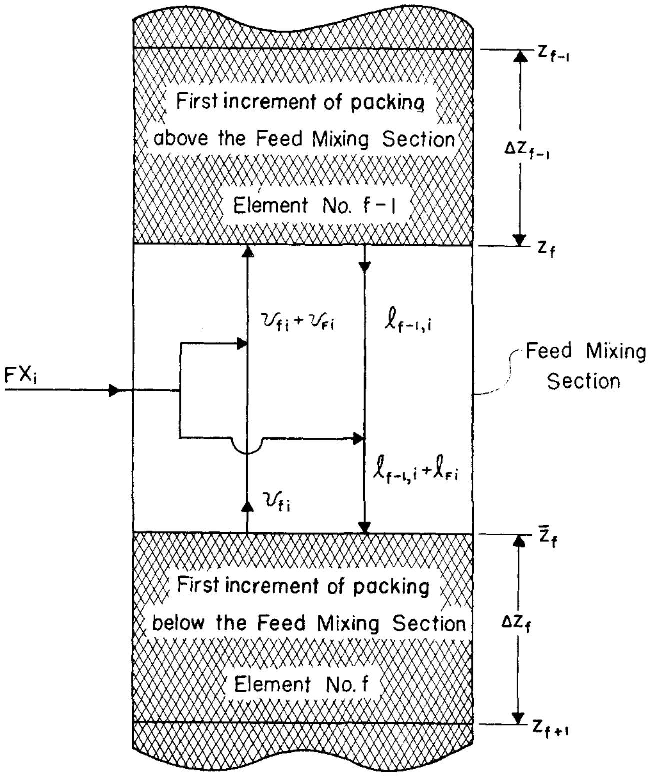

Fig. 2. The increments of packing immediately above and below the feed mixing section of a packed distillation column.

Then, the component-material balance enclosing any element Δz_j [given by the last expression of Equation (5)] may be rearranged to the following form by use of Equation (2),

$$A_{j-1,i}\,v_{j-1,i} - (1 + A_{ji})\,v_{ji} + v_{j+1,i} = 0 \qquad (3)$$

which is precisely the same expression as that obtained for plate j for a column with plates. In fact, the component-material balances for a packed column become identical with those for a column with plates, where for each element of packing Δz_j there exists a plate j. For example, consider the sketch of a typical packed column shown in Figure 1. In the column used to make the field tests, the section where the feed was introduced ($z_f \leqq z \leqq \bar{z}_f$) contained a liquid distributor but no packing as indicated in Figures 1 and 2. This section of the column is called the *feed mixing section.* The behavior of this portion of the column is approximated by making the supposition that the feed upon entering this section either flashes adiabatically at the column pressure or that it mixes perfectly with the liquid on the feed distributor plate. For the first case, it is further supposed that the liquid stream, L_F, formed by the flash mixes perfectly with the liquid stream L_{f-1} and enters the packing at $\bar{z}_f$. Likewise, it is supposed that the vapor stream, V_F, formed by the flash mixes perfectly with the vapor stream V_f and enters the packing at z_f. For the second case, the distributor plate is regarded as one mass transfer section and is included in the number N. When the component-material balances of the form of Equation (3) for each element of packing, the accumulator-condenser section, and the reboiler are stated in matrix notation, the matrix equation obtained is precisely the same as the one stated previously by Nartker, et al. (*12*) for distillation columns with plates. For any given set of temperatures (or K_{ji}^L's), a known set of E_{ji}'s, and a set of L_j/V_j's, the matrix equation so obtained is readily solved for the v_{ji}'s. Then the corresponding l_{ji}'s may be computed by use of Equation (2).

If a set of E_{ji}'s is known for a given number of elements of packing, the determination of the product distributions, temperature, and total flow rate profiles is carried out (except for minor variations) in a manner analogous to that summarized recently for plate-columns at steady state (*12*) and unsteady state operation (*8*).

Prior to the description of the analysis of the results of field tests, the relationships between the E_{ji}'s and the mass transfer coefficients as well as the HTU's are developed.

Rates of Mass Transfer

Let the rate of mass transfer of component i at any z of packing be denoted by N_i [moles of component i transferred/(unit time) (unit length of packing)]. Then, the rate of transfer of component i from the liquid to the vapor phase at any z may be expressed as follows:

$$N_i = (K_G a f^V)_i\, S[Y_i - y_i] \qquad (4)$$

where Y_i is used to denote $\gamma_i^L K_{ji}^L x_i$. In the interest of simplicity, Y_i is taken equal to $K_i^L x_i$ in the remainder of the development.

For each element of packing $\Delta z_j (j = 1, 2, \ldots, N)$, the component material balances are given by

$$\begin{aligned} v_{j+1,i} - v_{ji} + \int_{z_j}^{z_{j+1}} N_i\,dz &= 0 \\ l_{j-1,i} - l_{ji} - \int_{z_j}^{z_{j+1}} N_i\,dz &= 0 \qquad (5) \\ v_{j+1,i} + l_{j-1,i} - v_{ji} - l_{ji} &= 0 \end{aligned}$$

For elements of packing adjacent to the feed mixing section, the vapor and liquid flow rates entering the packing from the feed mixing section are modified as indicated in Figure 2. Note that of the three equations given by Equation (5), only two are independent for any given element of length Δz.

RELATIONSHIP BETWEEN THE VAPORIZATION EFFICIENCIES, MASS TRANSFER COEFFICIENTS, AND HEIGHT TRANSFER UNITS

The purpose of the following developments is to show that for a given choice of N there exists a unique set of vaporization efficiencies and to relate the vaporization efficiencies to the mass transfer coefficients or height transfer units.

The vaporization efficiency and the overall mass transfer coefficient for a given element of packing Δz_j are related in the following manner. By use of the generalized mean value theorem of integral calculus, it follows that the integral of N_i over Δz_j that appears in two of the component-material balances of Equation (5) may be restated as follows:

$$\int_{z_j}^{z_{j+1}} N_i\,dz = (\overline{K_G a f^V})_{ji}\, S \int_{z_j}^{z_{j+1}} (Y_i - y_i)\,dz \qquad (6)$$

By use of the trapezoidal rule, the integral appearing on the right-hand side of Equation (6) may be stated as follows:

$$\int_{z_j}^{z_{j+1}} N_i\,dz = (\overline{K_G a f^V})_{ji}\, S[(Y_{ji} - y_{ji}) + (Y_{j-1,i} - y_{j+1,i})]\,\frac{\Delta z_j}{2} + (\overline{K_G a f^V})_{ji}\, S\, Tr_{ji} \qquad (7)$$

where Tr_{ji} is the truncation error for the trapezoidal rule. Generally $Y_{ji} - y_{ji} \neq 0$. Let E_{ji} be that multiplier of Y_{ji} required to give

$$E_{ji} Y_{ji} - y_{ji} = 0$$

which may be rearranged to give the mass transfer relationship, Equation (1). Since the sum of the y_{ji}'s over all components is unity, the temperature function corresponding to the well known bubble point expression for a perfect plate (*7*) is obtained by summing each side of Equation (1) over all i followed by rearrangement to give

$$f_j = \sum_{i=1}^{c} E_{ji}\,\gamma_{ji}^L K_{ji}^L x_{ji} - 1 \tag{8}$$

Thus, the temperature T_j^L of the liquid phase is that absolute temperature $T_j^L > 0$ required to make $f_j = 0$.

Equation (7) and the second expression of Equation (5) may be combined to give

$$(\overline{K_G a f^V})_{ji} = \frac{[l_{j-1,i} - l_{ji}]}{S\,\dfrac{\Delta z_j}{2}\,[(1 - E_{ji})K_{ji}^L\,x_{ji} + K^L{}_{j-1,i}\,x_{j-1,i} - E_{j+1,i}K^L{}_{j+1,i}x_{j+1,i}]} \tag{9}$$

where the truncation error has been neglected. The corresponding value for the height of each transfer unit is given by

$$(\text{HTU})_{ji} = \frac{V_j}{(\overline{K_G a f^V})_{ji}\,S} \tag{10}$$

Determination of a Unique Set of Vaporization Efficiencies

Suppose that for a completely defined packed distillation column operating at a known set of steady state conditions, the additional specifications stated below are available from the results of field tests. By a completely defined column and a known set of operating conditions is meant that the following information is available: the diameter of the column, the type of packing, the depth of packing, the type of condenser (partial or total), the column pressure, the distillate rate D, the reflux rate L_O, the feed rate F as well as the composition and thermal condition of the feed. The additional specifications are as follows: x_{ji} $(0 \leqq j \leqq N + 1$, and $1 \leqq i \leqq c)$; X_{Di} $(1 \leqq i \leqq c)$; T_j^L $(0 \leqq j \leqq N + 1)$; and T_j^V $(0 \leqq j \leqq N + 1)$.

Although this particular set of specifications is seldom known in practice, the set is of interest because it yields a unique set of E_{ji}'s, which may be computed directly without resorting to the use of trial-and-error procedures.

First the L_j's are determined by use of enthalpy balances which require only the knowledge of the T_j^L's, T_j^V's, and x_{ji}'s. [The y_{ji}'s may be eliminated by use of the constant-composition method (*7*).] After the L_j's have been determined, the corresponding V_j's are found by use of the usual total-material balances (*7*).

On the basis of this set of total flow rates and the known values of the x_{ji}'s, T_j^L's, and T_j^V's, the component-material balances may be solved for the E_{ji}'s in the following manner. The component-material balance for any element Δz_j $(j \neq 0, f - 1, f, N + 1)$ may be solved for the unknown vapor rates to give

$$v_{j+1,i} - v_{ji} = -\,[l_{j-1,i} - l_{ji}] \tag{11}$$

The unknown vapor rates are stated in terms of the unknown efficiencies as follows:

$$v_{ji} = V_j\,y_{ji} = V_j\,E_{ji}\,K_{ji}^L\,x_{ji} = a_{ji}\,E_{ji} \tag{12}$$

where $a_{ji} = V_j\,K_{ji}^L\,x_{ji}$. Thus, the component-material balances for a conventional column with a partial condenser are represented by the following matrix equation.

$$\begin{bmatrix} -a_{0i} & a_{1i} & 0 & 0 & 0 & 0\,\cdot & \cdot & \cdot\,0 \\ 0 & -a_{1i} & a_{2i} & 0 & 0 & 0\,\cdot & \cdot & \cdot\,0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & -a_{Ni} & a_{N+1,i} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -a_{N+1,i} \end{bmatrix} \begin{bmatrix} E_{0i} \\ E_{1i} \\ \cdots \\ E_{Ni} \\ E_{N+1,i} \end{bmatrix} \begin{bmatrix} P_{0i} \\ P_{1i} \\ \cdots \\ P_{Ni} \\ P_{N+1,i} \end{bmatrix} \tag{13}$$

The following expression for E_{ji} is developed by applying the matrix multiplication rule to Equation (13) followed by the successive substitution of the expression for $E_{j+1,i}$ into the one for E_{ji}.

$$E_{ji} = -\frac{1}{a_{ji}} \sum_{k=j}^{N+1} P_{ki} \tag{14}$$

By direct substitution of each E_{ji} given by Equation (14) into the expression for the temperature function [Equation (8)], it is found that $f_j = 0$ for all j. Thus, this set of E_{ji}'s satisfy all equations describing the column.

After the set of E_{ji}'s has been determined, the corresponding unique set of mass transfer coefficients are found by use of Equation (9) and the height of each transfer unit by use of Equation (10). Thus, it is seen that for a given set of E_{ji}'s corresponding sets of overall mass transfer coefficients and height transfer units exist, provided of course it is permissible to apply the generalized mean value theorem as indicated by Equation (6), and all subsequent divisions are permissible. The vaporization efficiencies are also related to the Murphree plate efficiencies and the overall plate efficiencies as shown previously (*3*, *7*, *8*).

DETERMINATION OF VAPORIZATION EFFICIENCIES FROM THE RESULTS OF FIELD TESTS

Tests were made on a packed distillation column and a packed absorber at the Zoller Gas Plant at Refugio, Texas. This plant is used to recover propane and heavier hydrocarbons from natural gas feed stocks produced by the Zoller Gas Field and the Fulton Beach Gas Field. This plant has been described in detail by McDaniel (*11*). In order to measure the temperature profiles, eighteen iron-constantan thermocouples (15 in. in length) were inserted in the distillation column. The column consisted of two packed sections; the top section was 36 in. and the bottom section was 48 in. in diameter. The packing consisted of 2-in. Pall Rings. The absorber had a diameter of 36 in., and it was also packed with 2-in. Pall Rings. Six copper-constantan thermocouples, 15-in. in length, were inserted in the packing. The temperatures of the terminal streams were measured either with thermometers or thermocouples. Gas streams entering and leaving the columns were analyzed by use of a gas-phase chromatograph and the liquid streams were analyzed by means of low temperature distillations. The liquid stream (or bottoms) from the distillation column was also analyzed by means of a gas chromatograph. Various means of reducing these data as required to place the respective columns in component and total material balance were investigated, and the best of these were selected as described by Rubac (*16*) and McDaniel (*11*). The reduced results for typical tests for the distillation column and for the absorber are shown in Tables 1 and 2.

Determination of the Vaporization Efficiencies for the Distillation Column

In the analysis of the results of the field tests for packed

TABLE 1. REDUCED DATA FOR DISTILLATION COLUMN

Flow Rates, Moles/hr.	
F	405.9
D	158.9
L_0	237.4
feed temperature, (°F.)	360.0
condenser temperature, (°F.)	107.0
column pressure, (lb./sq.in.abs.)	165.0
Feed Composition, X_i	
ethane	0.0028
propane	0.1963
i-butane	0.0810
n-butane	0.0574
i-pentane	0.0230
n-pentane	0.0127
hexane	0.0130
heptane	0.0662
octane	0.1966
nonane	0.1604
decane	0.1906
Product Distributions, $[(b_i/d_i)_{co}]$	
propane	0.1240×10^{-3}
i-butane	0.2254×10^{-2}
n-butane	0.1274×10^{-2}
i-pentane	0.4251×10^{-2}
n-pentane	0.7211×10^{-2}
hexane	0.9275×10^{-1}
heptane	0.9065×10^{1}
octane	0.1195×10^{4}
nonane	0.4097×10^{5}

TABLE 2. REDUCED DATA FOR ABSORBER
Total Flow Rates

Lean Oil = 208.2457 lb. moles/hr.
Lean Gas = 1,414.3905 lb. moles/hr.
Rich Gas = 1,554.8367 lb. moles/hr.

Component	Component Flow Rates Lean Oil	Rich Gas
methane	0.0	1402.6870
ethane	0.0	96.9920
propane	0.0	35.3373
i-butane	0.0	9.0911
n-butane	0.0	6.8045
i-pentane	0.0461	1.4042
n-pentane	0.0631	0.8300
hexanes	2.6863	1.6906
heptanes Plus	205.4502	0.0

Component	Product Distributions l_{Ni}/v_{1i}
methane	$0.42371412 \times 10^{-1}$
ethane	0.48036497×10^{0}
propane	0.12158555×10^{2}

distillation columns, the vapor and liquid streams leaving each element of packing were taken to be at the same temperature ($T_j^L = T_j^V$), the temperature at the midpoint of each element of packing. A typical temperature profile which was determined experimentally is shown in Figure 3.

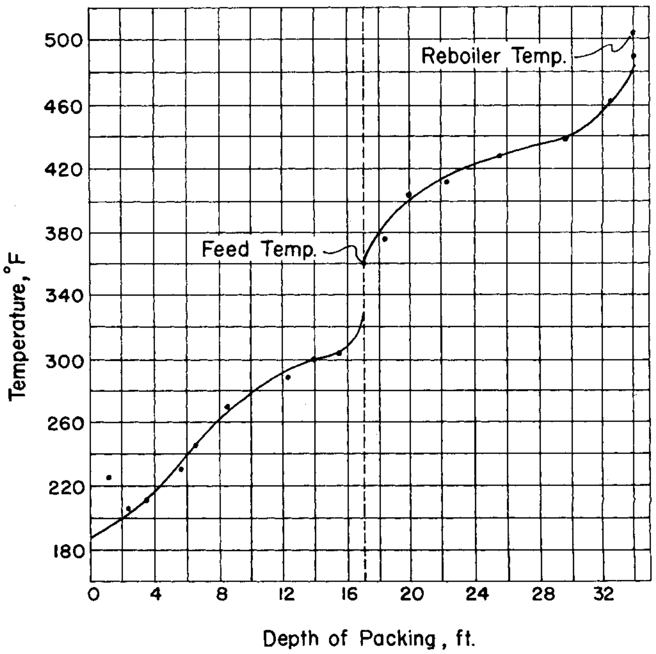

Fig. 3. Observed temperature profile in the packed distillation column.

There are jxi [or $c(N + 1)$] unknown values of E_{ji} appearing in the component-material balances. (For a column with a total condenser, which was the case, $E_{0i} = 1$.) If c product distributions, b_i/d_i, and $N + 1$ temperatures (the temperature of each element and the reboiler are known, then there exists $j + i$ (or $c + N+1$) known quantities with which to compute jxi values of E_{ji}. This problem is recognized as the same one solved previously by Taylor, et al. (*17*), Davis, et al. (*3*), and Holland (*7, 8*) in which the following model for the vaporization efficiencies was employed,

$$E_{ji} = \overline{E}_i \beta_j \tag{15}$$

where $\overline{E}_i$ is a component factor, and β_j is a section factor for element Δz_j. The calculational procedure employed makes use of the $j + i$ (or $N + 1 + c$) known temperatures and product distributions to compute c values of $\overline{E}_i$ and $N + 1$ values of β_j. For cases where b_i/d_i was unknown from test results, the corresponding $\overline{E}_i$ was set equal to unity as described by Davis, et al. (*3*). Because of the normalization procedure the final value differs slightly from unity (*3, 17*). If the temperature for any element Δz_j is regarded as unknown, the corresponding β_j is set equal to unity. In the analysis that follows, the feed mixing section was treated as a mass transfer section. The β was set equal to unity, and the temperature of the liquid of this element was computed by use of Equation (8). Except for this minor variation, the sets $\{\overline{E}_i\}$ and $\{\beta_j\}$ were computed by use of the calculational procedure proposed by Taylor, et al. (*17*) for each of several values of N. The K-values employed are given in Table 5* and (*11, 16*) and the enthalpy data are given in Table 6* and (*11, 16*).

It should be noted that the equations describing a packed distillation column are exact for any choice of the number of elements of packing equal to or greater than two. If one were interested in calculating the overall mass transfer coefficients corresponding to a given set of E_{ji}'s by use of Equation (9), then N should be selected such that the truncation error is less than some prescribed number. However, if the E_{ji}'s are available, the mass transfer coefficients are not needed to compute product distributions because the latter may be calculated by use of the same procedure as those described for columns with plates (*7, 8, 12*).

In general, it is desirable to simplify the model of the column in so far as possible by choosing N such that the parameters (the E_{ji}'s or the $\overline{E}_i$'s and the β_j's) are reduced

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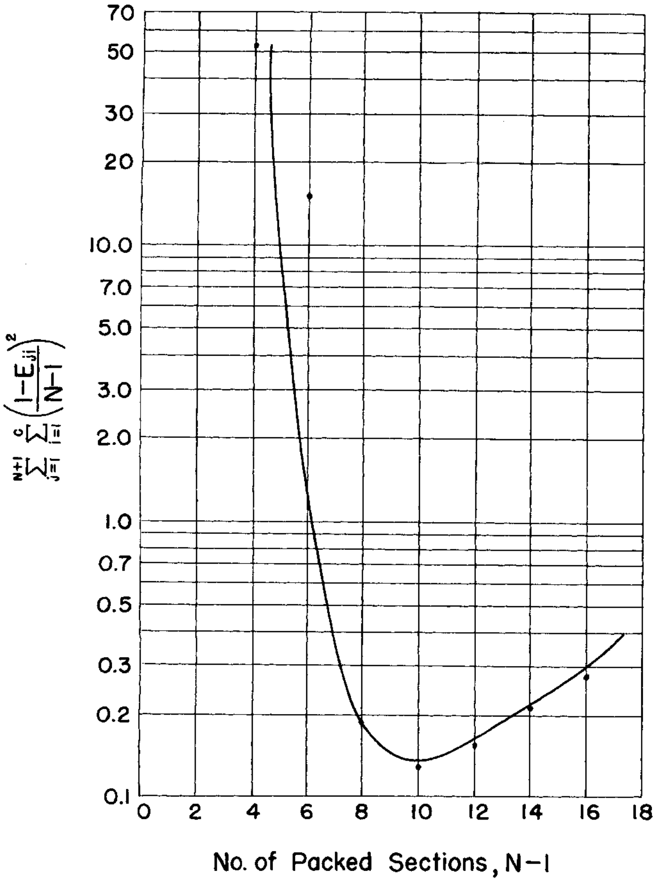

Fig. 4. Variation of $\sum_{j=1}^{N+1}\sum_{i=1}^{c}\left(\frac{1-E_{ji}}{N-1}\right)^2$ with the number of packed sections.

either to unity or to numbers which are as close to unity as is possible. More specifically, let

$$O(N) = \sum_{i=1}^{c}\sum_{j=1}^{N+1}\frac{(1-E_{ji})^2}{N-1} \tag{16}$$

Then the N that minimizes the function $O(N)$ is that N for which the deviation of all sections for each component from perfect plates is minimized. As shown in Figure 4, $O(N)$ was a minimum for $N - 1 = 10$ packed sections, which corresponds to a height of 3.4 ft. packed section. The corresponding $\overline{E}_i$'s and β_j's are presented in Table 3. Note that this height represents the closest approach to perfect plates or HETP's which can be achieved by dividing the packing into sections of equal length. An HETP of 2.8 ft./section for the bottom and an HETP of 2.5 ft./section for the top of the distillation column has been stated by Echert (*4*) for a column of this same description.

TABLE 3. $\overline{E}_i$'S AND β_j'S CALCULATED FOR $N = 11$ FOR THE DISTILLATION COLUMN

Component	$\overline{E}_i$	Element no.	β_j
propane	0.981	1	0.803
i-butane	1.292	2	1.276
n-butane	1.133	3	1.245
i-pentane	1.363	4	1.055
n-pentane	1.399	5	1.107
hexane	1.429	6*	0.999
heptane	1.247	7	0.760
octane	1.085	8	0.916
nonane	1.145	9	1.002
Components with unknown		10	1.053
Product Distributions	1.001		
		11	1.019
		12†	0.896

* Feed section.
† Reboiler.

Determination of the Vaporization Efficiencies for the Absorber

Absorbers have the general configuration shown in Figure 5 from which it is evident that the equations describing these units are of the same form as those for a packed section of a distillation column.

The results of field tests showed that the temperature profiles for absorbers are much different from those generally expected for distillation columns. Frequently, the scheme of operation of absorbers requires a crossover in

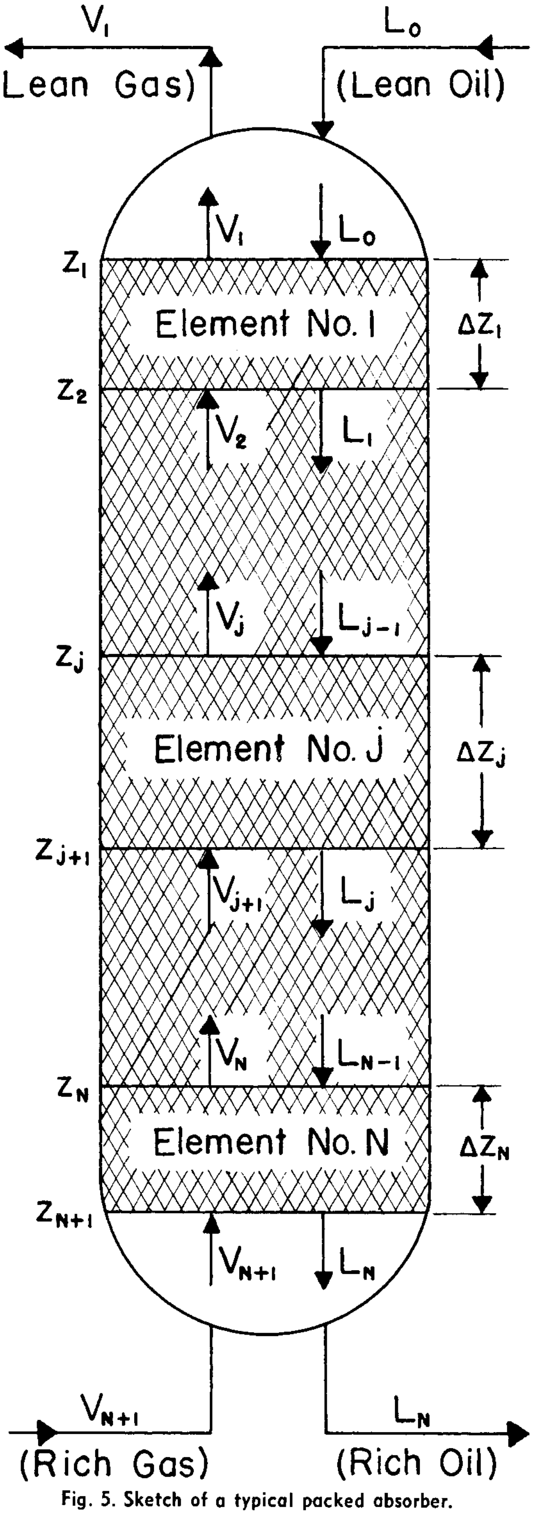

Fig. 5. Sketch of a typical packed absorber.

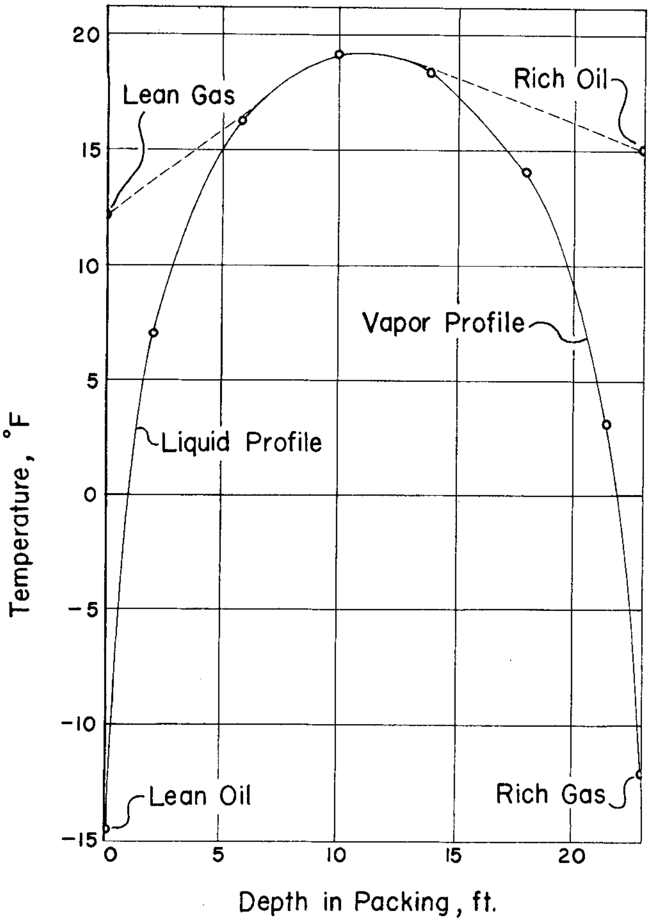

Fig. 6. Observed temperature profiles in the packed absorber.

the vapor and liquid stream temperatures. This crossover is illustrated by a typical field test which is portrayed in Figure 6. Since absorption is a heat liberating process, it is customary to introduce the rich gas and lean oil into the column at relatively low temperatures in order that the heat of absorption may be taken up by these streams. In all of the field tests, the temperature of the liquid phase was found to pass through a maximum as demonstrated in Figure 6. The experimental results are shown by points and a solid line. The terminal temperatures of all streams were measured, and temperatures measured near the bottom of the column near the inlet of the rich gas were taken to be vapor temperatures.

In the analysis of the experimental results, it was supposed that the crossover and the maximum values of T^V and T^L all occurred at the same z as indicated in Figure 6.

On the basis of these temperature profiles, the test results in Table 2, and the K-values and enthalpy data in Tables 5 and 6†, the vaporization efficiencies were determined for each of several values of N by use of the calculational procedures proposed by Davis, et al. (*3*). In this case, the procedure of setting the $\overline{E}_i$'s equal to unity for those components whose product distributions were unknown and accepting as the product distributions for these components those calculated at each N caused the $\overline{E}_i$'s for the components whose product distributions were known to increase with N. Thus, $O(N)$ tended to increase with N. However, it was observed that the β_j's were close to unity for most choices of N. The function

$$\sum_{j=1}^{N} (1-\beta_j)^2$$

oscillated between small numbers as N was varied, but did pass through a global minimum at $N = 8$. However, this global minimum was not well defined in that it was not appreciably smaller than other local minima. Since the β_j's did not deviate appreciably from unity, they were set equal to unity and the $\overline{E}_i$'s and liquid-temperature profiles were determined for each of several values of N. From a plot of these results shown in Figure 7, a well defined minimum in the deviation of the calculated and observed temperatures occurs at $N = 13$. A comparison of the calculated and observed temperature profiles for the liquid are shown in Figure 8. The $\overline{E}_i$'s corresponding to $\beta_j = 1$ (for all j) and the calculated temperature profile in Figure 8 are listed in Table 4. Also, the $\overline{E}_i$'s and β_j's required to obtain the observed temperature profile in Figure 7 (or 10) are also given in Table 4. An $N = 13$, corresponds to a height per section of 1.77 ft. Echert (*4*) gives an HETP of 2.8 ft. for a similar column.

TABLE 4. EFFICIENCIES FOR THE ABSORBER

I. $\overline{E}_i$'s Calculated for $N = 13$ with all β_j's Set Equal to Unity

Component	$\overline{E}_i$
methane	1.9088
ethane	1.2637
propane	1.2627
butanes and heavier	1.0000

II. $\overline{E}_i$'s and β_j's Calculated for $N = 13$

Component	$\overline{E}_i$	Element no.	β_j
methane	1.9606	1	1.2399
ethane	1.3080	2	1.0661
propane	1.3042	3	1.0147
butane and heavier	1.0571	4	0.9964
		5	0.9871
		6	0.9790
		7	0.9770
		8	0.9813
		9	0.9659
		10	0.9603
		11	0.9492
		12	0.9483
		13	0.9673

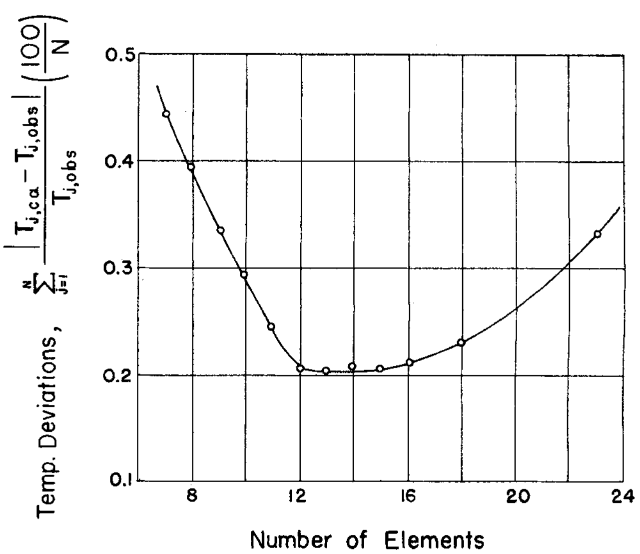

Fig. 7. Variation of the observed and calculated temperature profiles with the number of elements N, at $\beta_j = 1$ for all j.

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† See footnote page 571

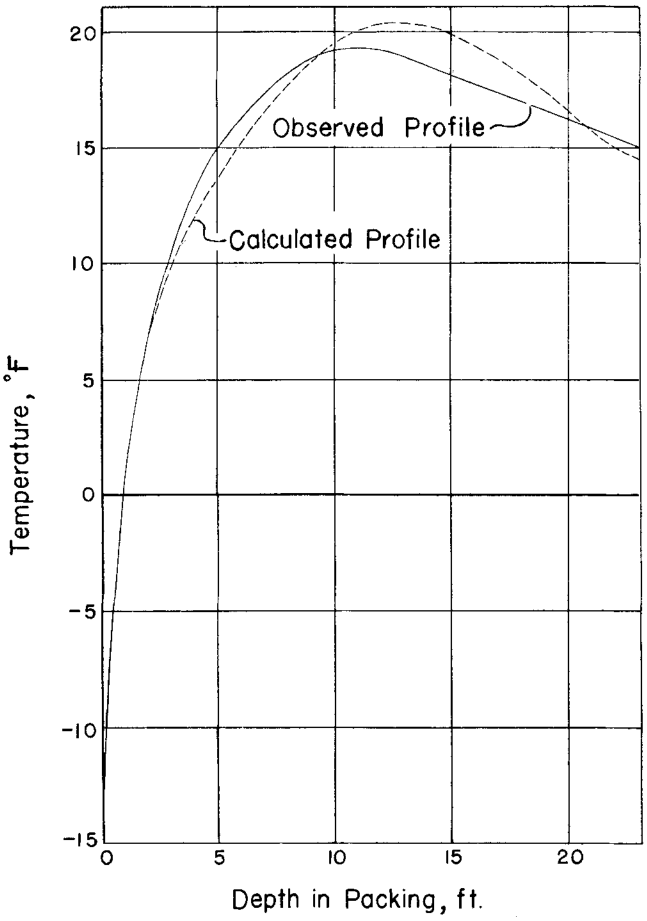

Fig. 8. Comparison of the observed and calculated liquid-temperature profiles.

CONCLUSIONS

The vaporization efficiencies constitute a unifying concept in that they provide calculational procedures for packed columns which are exactly the same as those for columns with plates. Furthermore, the vaporization efficiencies provide a relationship between HTU's (or mass transfer coefficients) and HETP's.

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NOTATION

a = interfacial area for mass transfer in sq.ft. of surface per cu.ft. of empty column

a_{ji} = $V_j K_{ji}^L x_{ji}$; $a_{0i} = D K_{0i} x_{0i}$

A_{ji} = $L_j/(E_{ji} \gamma_{ji}^L K_{ji}^L V_j)$; $A_{0i} = L_0/(E_{0i} \gamma_{0i}^L K_{0i}^L D)$; $A_{N+1,i} = B/(E_{N+1,i} \gamma^L_{N+1,i} K^L_{N+1,i} V_{N+1})$, modified absorption factors

b_i = molal flow rate of component i in the bottoms

B = total molal flow rate of bottoms

d_i = molal flow rate of component i in the distillate; also denoted by v_{0i}

D = total molal flow rate of bottoms

$\overline{E_i}$ = component efficiency, a function of i alone

E_{ji} = vaporization efficiency

f_j = temperature function for the jth mass transfer section

$f^V{}_{ji}$ = vapor fugacity of pure component i, evaluated at the total pressure and temperature of the vapor leaving the jth mass transfer section

F = total molal flow rate of the feed

HETP = height of packing equivalent to a theoretical plate

HTU = height of a transfer unit

k = counting integer

K_{ji}^L = equilibrium vaporization constant; evaluated at the temperature and pressure of the liquid leaving the jth mass transfer section

$(\overline{K_G a f^V})_{ji}$ = mean value of the product of the mass transfer coefficient, the interfacial area, and the fugacity for component i over element j

l_{ji} = molal flow rate at which component i in the liquid phase leaves the jth mass transfer section

l_{0i} = molal flow rate of component i in the liquid reflux

l_{Fi}, v_{Fi} = molal flow rates of component i in the liquid and vapor parts, respectively, of a partially vaporized feed. For bubble point liquid and subcooled feeds, $l_{Fi} = FX_i$ and $v_{Fi} = 0$. For dew point vapor and superheated feeds; $v_{Fi} = FX_i$ and $l_{Fi} = 0$

L_j = total molal flow rate at which liquid leaves the jth mass transfer section

N = total number of mass transfer sections, which includes the feed-distributor plate where it is treated as a mass transfer section, but excludes the reboiler and partial condenser if such a condenser is employed

N_i = moles of component i transferred from the liquid to the vapor phase per unit time per unit length of packing in the positive direction of z at any $z(z_1 \leqq z \leqq z_{N+1})$

$O(N)$ = a function of the set of vaporization efficiencies

P_{ji} = $-(l_{j-1,i} - l_{ji}) - C_{ji}$, $C_{ji} = 0 (j \neq f-1, f)$, $C_{f-1,i} = v_{Fi}$, $C_{fi} = l_{Fi}$; $P_{0i} = l_{0i}$; $P_{N+1,i} = -(l_{Ni} - b_i)$

S = internal cross-sectional area of the column

S_{ji} = $(E_{ji} \gamma_{ji}^L K_{ji}^L V_j)/L_j$, stripping factor for component i evaluated at the conditions of the liquid leaving the jth mass transfer section

T_j^L, T_j^V = temperatures of the vapor and liquid streams leaving the jth mass transfer section. When these temperatures are taken to be equal, the superscripts are omitted

v_{ji} = molal flow rate at which component i in the vapor phase leaves the jth mass transfer section

V_j = total molal flow rate of vapor leaving the jth mass transfer section

x_{ji} = mole fraction of component i in the liquid leaving the jth mass transfer section

x_{Bi} = mole fraction of component i in the bottoms

X_i = total mole fraction of component i in the feed, regardless of state

X_{Di} = total mole fraction of component i in the distillate, regardless of state

y_{ji} = mole fraction of component i in the vapor leaving the jth mass transfer section

Y_{ji} = $\gamma_{ji}^L K_{ji}^L x_{ji}$; γ_{ji}^L activity coefficient for component i in the liquid leaving the jth mass transfer section

z = depth of packed bed, measured down from the top of the column

β_j = mass transfer factor for the jth mass transfer section

Superscripts

L = to be evaluated at the bulk conditions of the liquid phase
V = to be evaluated at the bulk conditions of the vapor phase

Subscripts

f = packing boundaries adjacent to the feed mixing section
i = component number $(1 \leqq i \leqq c)$
j = integer for numbering the increments of the column; the first increment at the top of the column is assigned the number 1 and the last increment at the bottom, the number $N + 1$. At the top of the column where the packing begins $z = z_1$, and at the bottom where the packing ends, $z = z_{N+1}$. The condenser is assigned the number zero, and the reboiler the number $N + 1$
N = total number of elements of packing plus the feed-distributor plate where it is treated as a mass transfer section

Mathematical Symbols

$\sum_{i=1}^{c} x_i$ = sum over all values x_i $(1 \leqq i \leqq c)$
$\{x_j\}$ = set of all values x_j belonging to the particular set under consideration

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Copolymerization and Terpolymerization in Continuous Nonideal Reactors

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A simple means for predicting composition distributions in copolymers is used to illustrate the strong influence of segregation effects in the reactor. A recycle loop reactor can be used to produce uniform copolymers by narrowing the macroscopic composition distribution, but there is a limiting microscopic distribution which results from probabilistic effects.

This paper is aimed at the development of industrially feasible techniques for the production of homogeneous copolymers and terpolymers. Such multicomponent polymers are often required to meet specific physical and chemical properties which are unobtainable with simple homopolymers. However, production of good quality co- and terpolymers is generally more difficult and requires more careful polymerization techniques than are needed for homopolymers.

The properties of homopolymers are primarily determined by two factors:

1. The average molecular weight and the distribution of molecular weights about this average.
2. The extent of branching and cross linking.

These factors are of course important for multicomponent polymers, but a third major factor must also be considered:

3. The average chemical composition of the polymer and the distribution of compositions about this average.

The importance of this third factor may be illustrated by considering styrene-acrylonitrile copolymers. The pure homopolymers (polystyrene and polyacrylonitrile) are incompatible and a gross physical mixture of these materials gives a very brittle product which is hazy or even opaque. In a styrene-acrylonitrile copolymer the two constituents are combined within individual polymer molecules, giving a new chemical species rather than a physical mixture. Provided that the compositions of the individual molecules are fairly similar, the copolymer will be self-compatible, giving a reasonably tough, transparent, colorless product. If, however, there is a broad range of molecular compositions, a poor product will again result due to internal incompatibility (*5, 6*).

Two copolymer molecules formed from the same monomer mixture will generally have slightly different compositions due to random (probabilistic) variations on the molecular level. However, these differences are usually very small and such microscopic composition distributions can often be ignored. A far more serious cause for composition distributions is due to actual changes in monomer compositions. In a mixture of two monomers, one will usually polymerize at a faster rate than the other. Thus the mono-